A Specimen of Dynamics,

Toward Uncovering and Reducing to Their Causes Astonishing Laws of Nature Concerning the Forces of Bodies and Their Actions on One Another

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PART I

EVER SINCE we made mention of establishing a New Science of Dynamics, many distinguished persons have requested a fuller explanation of this doctrine in various places. Therefore, since we have not yet had the leisure to put a book together, we shall here present some things that can shed light on it, light that will perhaps even return to us with interest, if we elicit the views of those who join power of thought with refinement of expression. We acknowledge that their judgments will be welcome to us, and hope that they help advance the work. Elsewhere we urged that in corporeal things there is something over and above extension, in fact, something prior to extension, namely, that force of nature implanted everywhere by the Creator. This force does not consist in a simple faculty, with which the schools seem to have been content, but is further endowed with conatus or nisus,†161 attaining its full effect unless it is impeded by a contrary conatus. This nisus frequently presents itself to the senses and, in my judgment, is understood by reason to be everywhere in matter, even where it is not obvious to sense. But if we should not attribute this nisus to God, acting by miracle, then it is certainly necessary that he produce that force in bodies themselves, indeed, that it constitute the innermost nature of bodies, since to act is the mark of substances, and extension means nothing but the continuity or diffusion of an already presupposed striving and reacting (that is, resisting) substance. So far is extension from being able to constitute a substance itself! Nor does it matter that every corporeal action derives from motion, and that motion itself comes only from motion, either previously existing in the body or impressed from without. For, strictly speaking, motion (and likewise time) never really exists, since the whole never exists, inasmuch as it lacks coexistent parts. And furthermore, there is nothing real in motion but a momentary something which must consist in a force striving [nitente]†162 toward change. Whatever there is in corporeal nature over and above the object of geometry or extension reduces to this. And finally, this view takes both the truth and the doctrines of the ancients into consideration. Just as our age has already saved from scorn Democritus' corpuscles, Plato's ideas, and the Stoics' tranquility in light of the most perfect interconnection of things, so now we shall make intelligible the teachings of the Peripatetics concerning forms or entelechies, notions which seemed enigmatic for good reason, and were scarcely perceived by their own authors in the proper way. Furthermore, we think that it is necessary not to destroy this philosophy accepted for so many centuries, but to explain it in such a way that it can be rendered self-consistent (where this is possible) and, further, to illuminate it, and augment it with new truths.

This plan of study seems to me to be the one best suited both for judiciousness in teaching and for the benefit of students. It prevents us from appearing more eager to destroy than to build, and it

prevents the arrogance of bold minds from throwing us, daily, in our uncertainty, into perpetually changing our views; but rather, by restraining the whim of sects (which is encouraged by the empty glory of novelty) and by establishing doctrines with certainty, it enables the human race, at long last, to advance unhaltingly toward greater heights, no less in philosophy than in mathematics. For if you just omit the harsher things they say against others, there is usually much that is good and true in the writings of the distinguished ancients and moderns, much that deserves to be brought to light and deposited in the public treasury. And would that people chose to do this, rather than waste time with criticizing, which satisfies only their own vanity. Fortune has certainly favored me with certain novelties of my own, to such an extent that my friends constantly tell me to think about them alone. But nevertheless, many things others have done please me in a way, and I judge each in accordance with its worth, however it might vary. Perhaps this is because, by thinking about many things, I have learned not to despise anything. But we must now return to the proper path.

Active force (which might not inappropriately be called power [virtus], as some do) is twofold, that is, either primitive, which is inherent in every corporeal substance per se (since I believe that it is contrary to the nature of things that a body be altogether at rest), or derivative, which, resulting from a limitation of primitive force through the collision of bodies with one another, for example, is found in different degrees. Indeed, primitive force (which is nothing but the first entelechy) corresponds to the soul or substantial form. But, for that reason, it pertains only to general causes, which are insufficient to explain the phenomena. And so we agree with those who deny that we should appeal to forms when treating the individual and specific causes of sensible things. This is worth pointing out, so that when we restore forms, as if by birthright, in order to uncover the ultimate causes of things, at the same time, we don't seem to want to revive the verbal swordplay of the common schools. Nevertheless, a conception of forms is necessary for philosophizing properly, and no one can think that he sufficiently understands the nature of body unless he has turned his mind toward such things and understood that the crude notion of corporeal substance, which depends on the imagination alone and was carelessly introduced some years ago through an abuse of the corpuscular philosophy (by itself excellent and most true), is imperfect, not to say false. This can be established by the argument that since this notion of corporeal substance doesn't completely exclude inactivity or rest from matter, it cannot explain the laws of nature that regulate derivative force. Similarly, passive force is also twofold, either primitive or derivative. And indeed, the primitive force of being acted upon [vis primitiva patiendi] or of resisting constitutes that which is called primary matter in the schools, if correctly interpreted. This force is that by virtue of which it happens that a body cannot be penetrated by another body, but presents an obstacle to it, and at the same time is endowed with a certain laziness, so to speak, that is, an opposition to motion, nor, further, does it allow itself to be put into motion without somewhat diminishing the force of the body acting on it. As a result, the derivative force of being acted upon later shows itself to different degrees in secondary matter. But having distinguished and set out these general and basic considerations, considerations from which we learn that it is on account of form that every body always acts, and that it is on account of matter that every body is always acted upon and resists, we must now proceed deeper still and, in the doctrine of derivative forces [virtus] and resistance, deal with the extent to which bodies are empowered with different degrees of nisus, or the extent to which they offer resistance in various

ways. For it is to these notions that the laws of action apply, laws which are understood not only through reason, but are also corroborated by sense itself through the phenomena.

Therefore, by derivative force, namely, that by which bodies actually act on one another or are acted upon by one another, I understand, in this context, only that which is connected to motion (local motion, of course), and which, in turn, tends further to produce local motion. For we acknowledge that all other material phenomena can be explained by local motion. Motion is the continual change of place, and so requires time. However, just as a mobile thing in motion has motion in time, so too at any given moment it has a velocity, which is greater to the extent that more space is traversed in less time. Velocity taken together with direction is called conatus. Furthermore, impetus is the product of the bulk [moles] of a body and its velocity, whose quantity is what the Cartesians usually call quantity of motion, that is, the momentary quantity of motion; although, more accurately speaking, the quantity of a motion, which exists in time, of course, arises from the sum over time of the impetuses (equal or unequal) existing in the mobile thing, multiplied by the corresponding times.†163 In arguing with the Cartesians, though, we have followed their terminology. But to speak in a way not inappropriate for scientific use, just as we can distinguish the progress we are now making from the progress we have made or will make, considering our present progress as an increment or element of progress, or just as we can distinguish the present descent from descent already made, descent which it augments, so too we can distinguish the present or instantaneous element of motion from that same motion extended through a period of time, and call the former motio. And so, what is commonly called quantity of motion would be called quantity of motio.†164 Although we can be casual in our use of words after having a proper understanding of them, beforehand, however, we must use them with care so as not to be deceived through ambiguity.

Furthermore, just as the numerical value of a motion [motus] extending through time derives from an infinite number of impetuses, so, in turn, impetus itself (even though it is something momentary) arises from an infinite number of increments successively impressed on a given mobile thing. And so impetus too has a certain element from whose infinite repetition it can only arise. Consider tube AC rotating around the immobile center C on the horizontal plane of this page with a certain uniform speed, and consider ball B in the interior of the tube, just freed from a rope or some other hindrance, and beginning to move by virtue of centrifugal force.†165

Figure 4:

It is obvious that, in the beginning, the conatus for receding from the center, namely, that by virtue of which the ball B in the tube tends toward the end of the tube, A, is infinitely small in comparison with the impetus which it already has from rotation, that is, it is infinitely small in comparison with the impetus by virtue of which the ball B, together with the tube itself, tends to go from place D to (D), while maintaining the same distance from the center. But if the centrifugal impression deriving from the rotation were continued for some time, then by virtue of that very circumstance, a certain complete

centrifugal impetus (D) (B), comparable to the rotational impetus D (D), must arise in the ball. From this it is obvious that the nisus is twofold, that is, elementary or infinitely small, which I also call solicitation, and that which is formed from the continuation or repetition of elementary nisus, that is, impetus itself. Nevertheless, I wouldn't want to claim on these grounds that these mathematical entities are really found in nature, but I only wish to advance them for making careful calculations through mental abstraction.

From this it follows that force is also twofold. One force is elementary, which I also call dead force, since motion [motus] does not yet exist in it, but only a solicitation to motion [motus], as with the ball in the tube, or a stone in a sling while it is still being held in by a rope. The other force is ordinary force, joined with actual motion, which I call living force.†166 An example of dead force is centrifugal force itself, and also the force of heaviness [vis gravitatis] or centripetal force, and the force by which a stretched elastic body begins to restore itself. But when we are dealing with impact, which arises from a heavy body which has already been falling for some time, or from a bow that has already been restoring its shape for some time, or from a similar cause, the force in question is living force, which arises from an infinity of continual impressions of dead force. And this is what Galileo meant when he said, speaking enigmatically, that the force of impact is infinite in comparison with the simple nisus of heaviness.†167 But even though impetus is always joined to living force, we shall nevertheless show below that these two differ.

Living force in any aggregate of bodies must, again, be understood as twofold, namely total force or partial force, and partial force, again, is either relative or directive, that is, it either belongs to the parts or is common to the whole. Relative or proper force is that by which bodies contained in an aggregate can act on one another; directive or common force is that by which the aggregate itself can, in addition, act outside of itself. Moreover, I call it "directive" since the entire force embodied in the direction as a whole is conserved in this variety of partial force. If we imagine that the aggregate suddenly froze solid, having eliminated the motion of the parts with respect to one another, this force alone would remain. Whence the total absolute force consists of the relative and directive forces taken together. But these things will be better understood from the rules to be treated below.

So far as one can establish, the ancients had knowledge only of dead force, and this is what is commonly called mechanics, which deals with the lever, the pulley, the inclined plane (where accounts of the wedge and the screw belong), the equilibrium of bodies, and the like. There we treat only the first conatus of bodies acting on one another, before those bodies have received impetus through acting. And although one might, in a certain way, be able to transpose the laws of dead force over into living force, great caution is needed; those who confused force in general with the product of bulk [moles] and velocity because they discovered that dead force is proportional to that product were misled in just such a way. For, as we once warned, this fact †168 holds in this case for a special reason. For example, if different heavy bodies are falling, then at the very beginning of their motion, at least, the very descents or the very quantities of space traversed in descent, though, at that point, infinitely small or elementary, would be proportional to the speeds or to the conatus of descent. But once they have made some progress, and once living force has arisen, then the speeds acquired are no longer proportional to the spaces already traversed in descent (in terms of which force ought to be measured, as I once showed

and will later show more fully), but are proportional only to the sum of their own elements. Galileo began to deal with living force (under a different name, granted, and, indeed, under a different conception) and was the first to explain how motion arises from the acceleration of heavy bodies in fall. Descartes correctly distinguished velocity from direction and also saw that what results in the collision of bodies is that which least changes the prior state. But he did not calculate the least change properly, first changing the direction alone, then the velocity alone, whereas the change must be determined by both at the same time. But how this could be escaped him; since he focused on modalities rather than realities, things so heterogeneous seemed incapable of being compared or treated at the same time, not to speak of his other errors on this matter.†169

Honoratus Fabri, Marcus Marci, Joh. Alph. Borelli, Ignatius Baptista Pardies, Claudius de Chales, and other very acute men have made contributions to the theory of motion that should not be despised, but they have not avoided these fatal mistakes. Huygens, who illuminated our age with his excellent discoveries, seems to be the first person I know of to have arrived at the pure and clear truth on this matter, and the first to have freed this subject from paralogisms through certain laws he once published. Wren, Wallis, and Mariotte, gentlemen excellent in these studies, though, granted, in different ways, all obtained virtually the same rules. But their views of the causes are not the same, and thus even these gentlemen, outstanding as they are in these studies, do not always draw the same conclusions. And, to that extent, the true sources of this science (which we have established)†† have not yet been disclosed. Nor indeed, does everyone acknowledge what appears certain to me, that repercussion or reflexion arises from elastic force alone, that is, from resistance due to internal motion. Nor has anyone before us explained the notion of force. These matters have hitherto troubled the Cartesians and others who could not even grasp that the totality of motion or impetus (which they take to be quantity of force) might be different after a collision than it was before, because they believed that if that were to happen, the quantity of force would change as well.

In my youth, then believing (with Democritus and also with Gassendi and Descartes, who are among his followers on this question) that the nature of body consists in inert mass [massa] alone, I published a small book under the title A Physical Hypothesis,†170 in which I presented the theory of motion, both in abstraction from the organization of things in the world, and as it is connected with the organization of things in the world, a theory which, I see, pleased many distinguished people more than its insignificant worth deserved. There I established that if we assume such a notion of body, then every body entering into a collision gives its conatus to the body which receives it, that is, it gives its conatus to that which directly poses an obstacle as such. For, in the instant of collision, it tries †171 to proceed, and thus it tries to carry the receiving body along with it, and that conatus ought to have its full effect on the receiving body, unless hindered by a contrary conatus (for I then believed in the indifference of body to motion or rest), indeed, it ought to have its full effect even if hindered by a contrary conatus, since those different conatuses ought to be combined with one another. It was obvious that there is no reason why the body entering into the collision shouldn't attain the result toward which it strives, or why the receiving body shouldn't receive the entire conatus of the first. Therefore, it was obvious that the motion of the receiving body is the combination of its original motion with that it newly received, that is, the combination of its original motion with the conatus of the other body. From this, I further

showed that if we understand there to be in body only mathematical notions, size, shape, place, and their change, or if we understand there to be strivings [conatus] for change in the body only at the very moment of collision, without their being any ground [ratio] for metaphysical notions, namely, no ground for active power [potentia actrix] in the form and laziness [ignavia] or resistance to motion in the matter, and thus, if it were necessary for the outcome of a collision to be determined by the geometrical composition of conatus alone, as we explained earlier, then I showed that it ought to follow that the conatus of a body entering into a collision, however small it might be, would be impressed on the whole receiving body, however large it might be, and thus, that the largest body at rest would be carried off by a colliding body however small it might be, without retarding it at all, since such a notion of matter contains not resistance to motion, but indifference. From this it follows that it would be no more difficult to put a large body into motion than a small one, and thus, that there would be action without reaction, and that there could be no measure of power [potentia], since anything could prevail over anything else. Since there were also many other things of this sort which are contrary to the order of things and which are opposed to the principles of the true metaphysics, I then thought (indeed, correctly) that, by way of the organization of things [structura systematis], the Wisest Author of things had avoided the consequences that follow per se from the bare laws of motion derived from geometry.†172

But after I examined all of this more deeply, I saw what a systematic explanation of things consists in, and noticed that my earlier hypothesis about the notion of body was imperfect. I also noticed, through other arguments as well as this one, that one can establish that something should be posited in body over and above size and impenetrability, something from which the consideration of forces arises, and that by adding the metaphysical laws of this something to the laws of extension, the laws of motion that I called systematic arise, namely, that all change comes about by stages, that all action has a reaction, that a new force is not produced unless an earlier one is diminished, and therefore that a body that carries another off with it is always slowed by the one it carries off, and that there is neither more nor less power [potentia] in an effect than there is in its cause. Since this law does not derive from the notion of bulk [moles], it is necessary that it follow from something else inherent in bodies, indeed from force itself, which always maintains its same quantity, even if it is realized in different bodies.†173 Therefore, I concluded from this that, because we cannot derive all truths concerning corporeal things from logical and geometrical axioms alone, that is, from large and small, whole and part, shape and position, and because we must appeal to other axioms pertaining to cause and effect, action and passion, in terms of which we can explain the order of things, we must admit something metaphysical, something perceptible by the mind alone over and above that which is purely mathematical and subject to the imagination, and we must add to material mass [massa] a certain superior and, so to speak, formal principle. Whether we call this principle form or entelechy or force does not matter, as long as we remember that it can only be explained through the notion of forces.

Today certain distinguished men, seeing this very fact, namely, that the common notion of matter is unsatisfactory, summon God ex machina, and withdraw all force for acting from things, like a certain Mosaic Philosophy (as Fludd once called it); but I cannot agree. I certainly grant their observation that there is no genuine influx of one created substance into another, if one considers the matter with

metaphysical rigor, and I also admit freely that everything always proceeds from God through a continual creation. However, I believe that there is no natural truth in things whose explanation [ratio] ought to be sought directly from divine action or will, but that God has always endowed things themselves with something from which all of their predicates are to be explained. Certainly, it is agreed that God created not only bodies, but also souls, to which primitive entelechies correspond. But these things will be proven elsewhere, after their own grounds have been set out in greater depth.

However, even though I admit an active and, so to speak, vital principle superior to material notions everywhere in bodies, I do not agree with Henry More and other gentlemen distinguished in piety and ability, who use an Archaeus (unintelligible to me) or hylarchic principle even for dealing with the phenomena, as if not everything in nature can be explained mechanically, and as if those who try to explain everything mechanically are thought to eliminate incorporeal things, not without the suspicion of impiety, or as if it were necessary, with Aristotle, to attach intelligences to the rotating spheres, or as if one ought to say that the elements rise or fall by virtue of form, a concise, but useless doctrine. With these views, I say, I do not agree, and such a philosophy pleases me no more than that theology of certain men, who believed that Jupiter thundered and caused the snow to such an extent that they even defamed those who investigated more particular causes with the charge of atheism. In my opinion, the middle way in which one satisfies both piety and knowledge is the best. That is, we acknowledge that all corporeal phenomena can be derived from efficient and mechanical causes, but we understand that these very mechanical laws as a whole are derived from higher reasons. And so we use this higher efficient cause only in establishing general and distant principles. But once these principles have been established, then afterwards, whenever we deal with the immediate and specific efficient causes of natural things, we should take no account of souls or entelechies, no more than we should drag in useless faculties or inexplicable sympathies. For that first and most general efficient cause should not enter into the treatment of particulars, except insofar as we contemplate the ends which divine wisdom had in thus ordering things, so that we might lose no opportunity for singing his praises and for singing most beautiful hymns.

Indeed, one can even bring final causes to bear from time to time with great profit in particular cases in physics (as I showed with the clearly remarkable example of an optical principle, which that most celebrated Molyneux greatly applauded in his Dioptrics),†174 not only the better to admire the most beautiful works of the Supreme Author, but also in order that we might sometimes discover things by that method [via] that are either less evident or follow only hypothetically on the method of efficient causes. Perhaps philosophers have not yet sufficiently seen just how useful this is. In general, we must hold that everything in the world can be explained in two ways: through the kingdom of power, that is, through efficient causes, and through the kingdom of wisdom, that is, through final causes, through God, governing bodies for his glory, like an architect, governing them as machines that follow the laws of size or mathematics, governing them, indeed, for the use of souls, and through God governing for his glory souls capable of wisdom, governing them as his fellow citizens, members with him of a certain society, governing them like a prince, indeed like a father, through laws of goodness or moral laws. These two kingdoms everywhere interpenetrate each other without confusing or disturbing their laws, so that the greatest obtains in the kingdom of power at the same time as the best in the kingdom of

wisdom. But we had promised to establish here the general rules of effective forces, rules which we can then use in explaining particular efficient causes.

Next, I arrived at the true way of measuring forces, indeed, I arrived at the very same measure but in widely different ways, the one a priori, from a very simple consideration of space, time, and action (which I shall explain elsewhere), the other, a posteriori, namely, through measuring the force by the effect it produces in consuming itself.†175 By effect here I understand not any arbitrary effect, but one for which the force has to be expended, or one in which the force has to be consumed, an effect which one can therefore call violent. This kind of effect is unlike the effect a heavy body traversing a perfectly horizontal plane produces, since the same force always remains when such an effect is produced. Although we might have obtained this way of estimating forces of ours even from such an effect, properly called a harmless effect, so to speak, we shall set such effects aside for now. Moreover, I have chosen from among violent effects the one which is most conducive to homogeneous division, that is, the one most capable of being divided into similar and equal parts, as in the ascent of a body endowed with heaviness. For the elevation of a heavy body by two or three feet is precisely double or triple the elevation of the same body by one foot, and the elevation of a heavy body, double in size, by one foot is exactly double the elevation of a single heavy body to a height of one foot. As a result, the elevation of a heavy body, double in size, by three feet is precisely six times the elevation of a single body by one foot. This assumes, of course, that the heavy bodies are unchanged in weight when more or less distant from the horizon, an assumption we can make for the sake of exposition, at least, even if things are, perhaps, different in reality, though the error here is insensible. On the other hand, homogeneity is not as easily found in an elastic body. Then, when I wanted to compare different bodies, or bodies endowed with different speeds, I easily saw that if body A were single and body B double in size, and both had the same speed, then the one would have a single unit of force, and the other two units, since, in the other body, there is precisely twice of whatever there is in the one, for B is a body twice the size of A, moving with equal speed, and nothing more.†176 But if A and C are equal in size, and A has a single unit of speed, and C has two units, I saw that C does not have precisely twice of whatever there is in A, since it is speed and not the size of the body that is doubled, of course. And I saw that those who believed that force itself is doubled by the mere doubling of a modality made a mistake at this point.†177 In just this way I observed and warned long ago that the true art of measuring, hitherto untreated, despite the fact that numerous books on the Universal Elements of

Figure 5: [p. 128]

Mathematics have been written, consists in arriving, at last, at something homogeneous, that is, at an exact and complete repetition not only of modes, but also of realities. One can give no better illustration or example of this method than what this very argument shows.

Therefore, in order to obtain a measure of force, I considered whether those two bodies A and C, equal in size but different in speed, could produce any effects equal in power to their causes, and

homogeneous with each other. For things which cannot easily be compared directly can at least be compared accurately through their effects. Moreover, I assumed that an effect ought to be equal to its cause, if that effect is produced by expending or consuming the entire power [virtus] of that cause; in this circumstance, it does not matter how much time it takes to produce the effect. Therefore, let us assume that bodies A and C are heavy, and that their force is converted into ascent, which would come about if, at the very moment when they had the speeds they were said to have, a single unit of speed in A, and double that in C, they were understood to be at the ends of the vertical pendula PA1 and EC1.†178 Now, it is well known from the demonstrations of Galileo and others that if body A, with a speed of one unit, ascends at its highest point above the horizon HR to a height A2H of one foot, then body C, with a speed of two units, could ascend (at its highest) to a height C2R of four feet. From this it already follows that a heavy body with two units of speed has four times as much power as a heavy body with one unit of speed, since by expending its entire power, it can bring about exactly four times the effect. For raising a pound (that is, raising the body itself) four feet raises one pound one foot precisely four times. In the same way one can conclude, in general, that the forces in equal bodies are proportional to the squares of their speeds, and thus that in general the forces in bodies are jointly proportional to the size of the bodies and the squares of the speeds.

I have confirmed this by reducing to absurdity (indeed, to perpetual motion) the contrary view commonly accepted, especially among the Cartesians, according to which forces are believed to be jointly proportional to the size of a body and its speed. I have also used this method repeatedly to define a posteriori two states of unequal power [virtus] and, at the same time, to find a sure mark for distinguishing greater power from less. For, when perpetual mechanical motion or an effect that is greater than its cause arises from substituting one thing for another, these states are hardly of equal power. Rather, that which was substituted for the other was more powerful since it caused something greater to appear. Moreover, I take it to be certain that nature never substitutes things unequal in their forces for one another, but that the entire effect is always equal to the full cause. And in turn, we can safely substitute things equal in force for one another in our calculations with complete freedom, just as if we had made that substitution in actuality, with no fear of perpetual mechanical motion arising as a consequence. Thus, if it were true, as people are commonly persuaded, that a heavy body A, two units in size (as we might now assume) and endowed with one unit of speed, and a heavy body C, one unit in size and endowed with two units of speed, are equal in power, then we should safely be able to substitute the one for the other. But this is not the case. For let us suppose that A, two units in size, has acquired its one unit of speed by descending along the path A2A1, from a height A2H of one foot, and at the moment it was at A1, that is, on the horizontal, let us substitute for it the weight C, one unit in size, two units in speed, equal in power, as they would have it, which ascends to C2 †† or to a height of four feet. And so, merely through the fall of a two-pound weight A from a height A2H of one foot, and by substituting for this something equal in power, we would have brought about the ascent of a one-pound body by four feet, which is double the power of the prior state. Thus we would have gained as much force as we started with, that is, we would have brought about perpetual mechanical motion, which is absurd. It does not matter whether we can actually bring this substitution about through the laws of motion, for we are safely able to substitute things equal in power for one another, even mentally. However, I have thought up various ways by which one can, as conveniently as you like, actually bring it

about that the total force of a body A is transferred to body C, which was previously at rest but which now (having reduced A to rest) is alone in motion.†179 Hence, it could happen that a one-pound weight with two units of speed takes the place of a two-pound weight with one unit of speed, if they were equal in power; but from this we showed that an absurdity arises. These questions are not empty, nor is this a mere argument over words; on the contrary, these things are of the greatest utility for comparing machines and motions. For if anyone were to have, from water or animals or some other cause, force enough to keep a heavy body of one hundred pounds in constant motion, by which it could complete a horizontal circle thirty feet in diameter in a quarter of a minute of time, and if another person, in its place, were to offer force enough to double the weight to complete only half the circle in the same time, with less expenditure of force, and reckoned this advantageous to you, he would have deceived you and shrewdly tricked you out of half of your force. But now, having eliminated the errors, let us set forth the true and, indeed, wonderful laws of nature a bit more distinctly in the second part of this sketch.

Part II

THE FACT that the nature of body, indeed of substance in general, is not known sufficiently well (as I have already touched upon) has brought it about that the distinguished philosophers of our times, locating the notion of body in extension alone, are thus, forced to appeal to God for explaining the union between soul and body, and indeed for explaining the interaction of bodies with one another. For we must admit that it is impossible that bare extension, containing geometrical notions alone, is capable of action and passion. And so, this one position seemed to be the only one left for them, that when a person thinks and tries to move his arm, God moves the arm for him as if by primeval agreement, and, conversely, that when there is motion in the blood and [animal] spirits, God excites a perception in the soul. But these very consequences, so foreign to correct reasoning in philosophy, ought to have warned these writers that they were depending on a false principle, and that the notion of body from which such consequences were derived had been improperly explicated. Therefore, we have shown that there is a force of acting in every substance, and that there is also a force of being acted upon [patiendi] in every created substance, and that the notion of extension is incomplete in itself, but is relative to something which is extended, something whose diffusion or continuous repetition extension indicates; further, we have shown that the notion of extension presupposes the substance of body, which involves the power of acting and resisting, and exists everywhere as corporeal mass [massa], and that the diffusion of this substance is contained in extension. From this we shall, at some later time, shed new light on the explanation of the union of the soul and the body. But now we must show how, from this, follows wonderful and most useful practical theorems in dynamics, that is, the science which deals chiefly with the laws [regulae] governing corporeal forces.

We must realize, above all, that force is something absolutely real in substances, even in created substances, while space, time, and motion are, to a certain extent, beings of reason, and are true or real, not per se, but only to the extent that they involve either the divine attributes (immensity, eternity, the

ability to carry out works), or the force in created substances. From this it immediately follows that there is no empty place and no empty moment in time. Moreover, it follows that motion taken apart from force, that is, motion insofar as it is taken to contain only geometrical notions (size, shape, and their change), is really nothing but the change of situation, and furthermore, that as far as the phenomena are concerned, motion is a pure relation, something Descartes also recognized when he defined motion as the translation from the neighborhood of one body into the neighborhood of another. But in drawing consequences from this, he forgot his definition and set up the laws of motion as if motion were something real and absolute. Therefore, we must hold that however many bodies might be in motion, one cannot infer from the phenomena which of them really has absolute and determinate motion or rest. Rather, one can attribute rest to any one of them one may choose, and yet the same phenomena will result. From this follows something that Descartes did not notice, that the equivalence of hypotheses is not changed even by the collision of bodies with one another, and thus, that the laws of motion must be fixed in such a way that the relative nature of motion is preserved, so that one cannot tell, on the basis of the phenomena resulting from a collision, where there had been rest or determinate motion in an absolute sense before the collision. As a result, Descartes's law, the law in accordance with which he holds that a body at rest cannot in any way be moved from its place by another smaller body, is hardly adequate, nor are other things of this sort, which are as far from the truth as one can go.†180 It also follows from the relative nature of motion that the mutual action or impact of bodies on one another is the same, provided that they approach one another with the same speed. That is, if we keep the appearances in the given phenomena constant, then whatever the true hypothesis might finally be, to whichever body we might in the end truly ascribe motion or rest, the same outcome would be found in the phenomena in question, that is, the same outcome would be found in the resulting phenomena, even as regards the action of bodies on one another. And indeed, this is just what we experience, for we would feel the same pain whether we hit our hand against a stone at rest, suspended, if you like, from a string, or whether the stone hit our resting hand with the same speed. However, we speak as the situation requires, in accordance with the more appropriate and simpler explanation of the phenomena. It is in just this sense that we use the motion of the primum mobile in spherical astronomy, while in the theoretical study of the planets we ought to use the Copernican hypothesis. As an immediate consequence of this view, those disputes conducted with such enthusiasm, disputes in which even the theologians were involved, completely disappear.†181 For even though force is something real and absolute, motion belongs among phenomena and relations, and we must seek truth not so much in the phenomena as in their causes.

Figure 6: [p. 131]

It also follows from our notions about bodies and forces that what happens in a substance can be understood to happen of that substance's own accord, and in an orderly way. Connected to this is the fact that no change happens through a leap. Assuming this, it also follows that atoms cannot exist. In order to grasp the force of this consequence, let us assume that bodies A and B collide as in figure 6, so that A1 arrives at A2 and likewise B1 arrives at B2, and so colliding at A2B2, they are reflected from A2 to A3 and from B2 to B3. However, if we were to imagine that there are atoms, that is, bodies of

maximal hardness and therefore inflexible, it would follow that there would be a change through a leap, that is, an instantaneous change. For at the very moment of collision the direction of the motion reverses itself, unless we assume that the bodies come to rest immediately after the collision, that is, lose their force; beyond the fact that it would be absurd in other ways, this contains, again, a change through a leap, an instantaneous change from motion to rest, without passing through the intermediate steps.

Figure 7: [p. 132]

And so, we must acknowledge that if bodies A and B collide as in figure 7, and come from A1 and B1 to the place A2B2, where they collide, they will, little by little, be compressed there, just like two inflated balls, and approach one another more and more, continually increasing the internal pressure. By that very circumstance the motion itself is weakened, the force of the conatus having been transformed into their elasticity, until they are altogether at rest. Then, finally, restoring themselves through their elasticity, they rebound from one another; having started a retrograde motion from rest, a motion that continually increases, in the end they move apart, having regained the same speed with which they originally approached one another, but directed oppositely, and they return to A3 and B3, which coincide with the places A1 and B1, if the bodies are assumed to be of the same size and the same velocity. From this it is already obvious how no change happens through a leap; rather, the forward motion diminishes little by little, and after the body is finally reduced to rest, the backward motion at last arises. In just the same way one shape does not arise from another (for example, an oval from a circle), unless it passes through innumerable intermediate shapes, nor does anything pass from one place to another, or from one time to another except by passing through all of the intermediate places and times. And so, rest will not arise from motion, much less will motion in an opposite direction arise, unless a body passes through all intermediate degrees of motion. Since this is of such importance in nature, I am amazed that it has been so little noticed. From this follows something Descartes opposed in his letters,†182 something many gentlemen of great reputation are even now unwilling to admit, that all rebound arises from elasticity, which explains many elegant experiments that show that a body is deformed before it is impelled, as Mariotte nicely demonstrated. And finally, a most wonderful conclusion follows from this, that no body is so small that it is without elasticity, and furthermore, each body is permeated by a fluid even subtler than it is. And thus, there are no elements of bodies, nor is there maximally fluid matter, nor are there little solid globes (unintelligible to me) of the second element, both determinate in shape and hard. Rather, the analysis proceeds to infinity.

It is also in agreement with that law of continuity, or the law excluding a leap in changing, that the case of rest can be considered as a special case of motion, indeed, the case of vanishing or minimal motion, and that the case of equality can be considered as a case of vanishing inequality. From this it follows that the laws of motion ought to be formulated in such a way that there is no need for special laws for equal bodies and bodies at rest. Rather, these laws arise per se from the laws of [un]equal

bodies and motions, or, if we want to formulate special laws for rest and equality, we must be careful not to formulate laws that are inconsistent with the hypothesis that takes rest as the limit of motion or equality as the least inequality, otherwise we will violate the harmony of things, and our laws will not be consistent with one another. I first published this new tool for examining our laws and those of others in the Nouvelles de la république des lettres†183 in July 1687, article 8, and called it a general principle of order that arises from the notions of infinity and continuity, something that suggests the axiom that, as the givens are ordered, so is that which is sought. I expressed the matter in a general way as follows: if one case continually approaches another case among the givens, and finally vanishes into it, then it is necessary that the outcomes of the cases continually approach one another in that which is sought and finally merge with one another. This is just as it is in geometry, where the case of the ellipse continually approaches that of the parabola; fixing one focus, if we assume the other to be moved farther and farther away, finally, in the case where the other focus is infinitely distant, the ellipse disappears into a parabola. From this it follows necessarily that all of the laws of the ellipse hold for the parabola, taken as an ellipse whose other focus is infinitely distant. And so, we can conceive of parallel rays intersecting a parabola as if they came from the other focus [at infinity] or proceed toward it. Therefore, the case in which body A collides with the moving body B can be continuously varied so that, holding the motion of A fixed, the motion of B is assumed to be smaller and smaller, until finally it is assumed to vanish into rest, and then increase once again in the opposite direction. I say that, in the same way, the outcome of the collision, or that which results either in A or in B, when both are in motion, continuously approaches the outcome of the collision that results when B is at rest, until finally the one case disappears into the other. Thus the case of rest, both in the givens and in the outcomes (that is, in that which is sought), is the limit of the cases of motion in a straight line, or, the common limit of continuous rectilinear motion, and thus, it is a special instance of it. When I examine the Cartesian laws of motion with respect to this touchstone, which I transposed from geometry into physics, it happens, much to my surprise, that a certain gap or leap, entirely abhorrent to the nature of things, displays itself. For, representing quantities by lines,†184 and taking the motion of B before the collision as the given case, represented in the abscissa, and its motion after the collision as the outcome sought, represented in the ordinate, and extending a line from one end of the ordinate to the other in accordance with Descartes's laws, this line was not a single continuous line, but was something wondrously gaping and leaping in an absurd and incomprehensible way. And since I noted on that occasion that even the laws proposed by the Rev. Father Malebranche do not entirely bear up under this examination, the distinguished gentleman, having considered the matter a second time, confessed in all candor that this is what gave him the occasion for altering the laws, which he presented to the public in a little book. However, one must admit that because he has not yet sufficiently attended to the use of this new tool, there remain things which even now do not sufficiently square up in every respect.†185

Something else wonderfully follows from what I have said, that every passion of a body is of its own accord, that is, arises from an internal force, even if it is on the occasion of something external. I understand here the body's own passion, the passion that arises from collision, that is, the passion that remains the same, whichever hypothesis we finally adopt, that is, to whatever things we ascribe absolute rest or motion in the end. For, since the impact is the same, wherever the true motion in the end belongs, it follows that the effect of the impact is equally distributed between the two, and thus

that in impact, both bodies are equally acted upon, and equally act, and that half the effect arises from the action of the one, and half from the action of the other. And since half the effect or half of the passion is in one, and half in the other, it is also sufficient for us to derive the passion in the one from its own action, and we do not need any influx of the one into the other, even if the action of the one provides the occasion for the other to produce a change in itself. For example, in Figure 7, when A and B collide, the resistance of the bodies joined with their elasticity results in their being compressed on account of the impact, and there is equal compression in both and according to any hypothesis. Experience also shows this. If one imagines that two inflated balls collide, then whether both are in motion, or one of the two is at rest, or even if the body at rest is suspended from a string so that it can rebound as easily as possible, as long as the speed with which they approach one another, that is, the relative speed, is always the same, the compression or the intensity of the elasticity will also be the same and equal in both. Furthermore, when balls A and B restore themselves through the force of their considerable elasticity, that is, through the force of the compression they contain, they will repel one another and burst forth as if from a bow, and each will repel itself from the other through forces equal to one another; thus each body will rebound, not through the force of the other, but through its own. What we have said about inflated balls must also be understood to apply to every body insofar as it is acted upon in impact, namely, that the repercussion and bursting apart arises from the elasticity it contains, that is, from the motion of the fluid aetherial matter permeating it, and thus it arises from an internal force or a force existing within itself. I understand here the proper motion of bodies (as I have called it) as opposed to the common motion that can be ascribed to the center of gravity. As a result, we should conceive of their proper motion (conceive of it in a hypothetical way, I say) as if the bodies are being carried on a boat which has the motion of their common center of gravity, and on the boat, we should imagine, they move in such a way that from the composite motion of the boat, which they have in common (that is, the motion of the center of gravity), and from their own proper motion, the phenomena can be saved. From what we have said we can also understand that the action of bodies is never without reaction, and both are equal to one another, and directed in opposite directions.

Also, since only force and the nisus arising from it exist at any moment (for motion never really exists, as we discussed above), and since every nisus tends in a straight line, it follows that all motion is either rectilinear or composed of rectilinear motions. From this it not only follows that what moves in a curved path always tries [conari] to proceed in a straight line tangent to it, but also--something utterly unexpected--that the true notion of solidity derives from this. (Nothing is really solid or fluid, absolutely speaking, and everything has a certain degree of solidity or fluidity; which term we apply to a thing derives from the predominant appearance it presents to our senses.) For if we assume something we call solid is rotating around its center, its parts will try [conabuntur] to fly off on the tangent; indeed, they will actually begin to fly off. But since this mutual separation disturbs the motion of the surrounding bodies, they are repelled back, that is, thrust back together again, as if the center contained a magnetic force for attracting them, or as if the parts themselves contained a centripetal force. Thus, the rotation arises from the composition of the rectilinear nisus for receding on the tangent and the centripetal conatus among the parts. Thus, all curvilinear motion arises from rectilinear nisuses composed with one another, and at the same time, it is understood that all solidity is caused by surrounding bodies pushing a body together; if matters were otherwise, then it could not happen that

all curvilinear motion is composed of pure rectilinear motions. From this we also get a new argument against atoms that is no less unexpected than the one before. Moreover, nothing can be imagined farther from things than the claim that solidity derives from rest, for there is never any true rest in bodies, nor can anything come from rest but rest.†186 Let us grant that A and B are at rest with respect to one another, if not truly at rest, then at least relatively (although this couldn't ever exactly happen, for no body preserves exactly the same distance from another no matter how short the time elapsed) and let us grant that whatever is at rest at one time will always be at rest, unless put into motion by a new cause. But it does not follow on that account that, since B resists being impelled, it also resists being separated from the other body, so that if one were to overcome the resistance of B, that is, if one were to put B into motion, A would immediately follow. But if there were attraction, something not found in nature, attraction explained either from primitive solidity, or through rest or something similar, then this would certainly have followed. And so, we should not explain solidity except through the surrounding bodies pushing a body together. For [mutual] pressure alone is insufficient to explain the matter, as if only A hindered the departure of B. But we must understand that the bodies do indeed separate from one another. However, one is driven back to the other by the surrounding bodies, and thus from the combination of these two motions the conjunction is preserved. And so, those who imagine certain planks or insensible plates in bodies (on the model of two polished marble plates which are carefully placed on one another),†187 planks or plates whose separation is made difficult because of the resistance of the surrounding bodies, and from this explain the solidity of hard, sensible bodies, even if they might often be right, these people don't give the ultimate explanation of solidity since they assume some solidity in the plates. From this we can also understand why, on this matter, I cannot agree with certain philosophical opinions of certain important mathematicians, who, beyond the fact that they admit empty space and don't seem to shrink from attraction, also take motion to be an absolute thing, and strive to prove this from rotation and the centrifugal force that arises from it.†188 But since rotation also arises only from a combination of rectilinear motions, it follows that if the equivalence of hypotheses is preserved in rectilinear motions, however they might be placed in things, then it will also be preserved in curvilinear motions.

From what was said one can also understand that motion common to many bodies does not change their actions with respect to one another, since the speed with which they approach one another, and therefore the force of impact by which they act on one another, is not altered. And from this follows those excellent experiments Gassendi related in his letters on motion impressed by a mover in motion in order to satisfy those who thought they could infer from the motion of projectiles that the earth is at rest.†189 However, it is certain that if some people were carried on a large boat (enclosed, if you please, or, at least, set up in such a way that things outside of the boat could not be seen by the passengers), then, however great the speed of the boat might be, as long as it moved peacefully and evenly, they would have no criterion for discerning (on the basis of that which happens on the boat, of course) whether the boat was at rest or in motion, even if they played vigorously with a ball or produce other motions. And this must be noted for the sake of those who, incorrectly understanding the opinion of the Copernicans, believe that according to them, things projected from the earth into the air are carried around by the air rotating with the earth, and because of that, follow the motion of the ground and fall back to earth as if they were at rest. This is correctly judged to be insufficient. However, the

most learned gentlemen who use the Copernican hypothesis conceive that whatever is on the surface of the earth moves with the earth, and thus, when something is shot out of a bow or a catapult †† it carries the impetus impressed by the rotation of the earth together with the impetus impressed by projecting it. As a result, since the motion of projectiles is twofold, one common with the earth, the other pertaining to the projection, it is not surprising that the common motion changes nothing. However, we must not hide the fact that if projectiles can be thrown far enough, or if we imagine a boat large enough, moving with sufficient speed, so that before the descent of the heavy body, the earth or the boat would describe an arc notably different from the rectilinear, then a criterion for discriminating motion and rest could be found, since then, indeed, the (circular) motion of the earth or ship would not remain in common with the (rectilinear) motion impressed on the missile by the rotation of the boat or the earth. And in addition, the nisus heavy bodies have toward the center [of the earth] adds an external action that can produce a distinction within the phenomena no less than if, enclosed in the ship, one had a compass that pointed to the pole, something that, at very least, would indicate when the boat turned. But, whenever we are dealing with the equivalence of hypotheses, we must take into account everything relevant to the phenomena. From these things we also understand that we can safely apply the composition of motions, or the resolution of one motion into two or however many more motions, even though a certain most ingenious gentleman expressed doubts about this in the presence of Wallis, and not absurdly. For the matter warrants a proof, in any case, and it cannot be assumed as if it were self-evident, as many have done.